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Investigation on CTH Detector Response using Simulation Software

Fitting Moyal and Landau functions onto p and dE/dx distributions therefore analysing using Bethe-Bloch formula

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Outline

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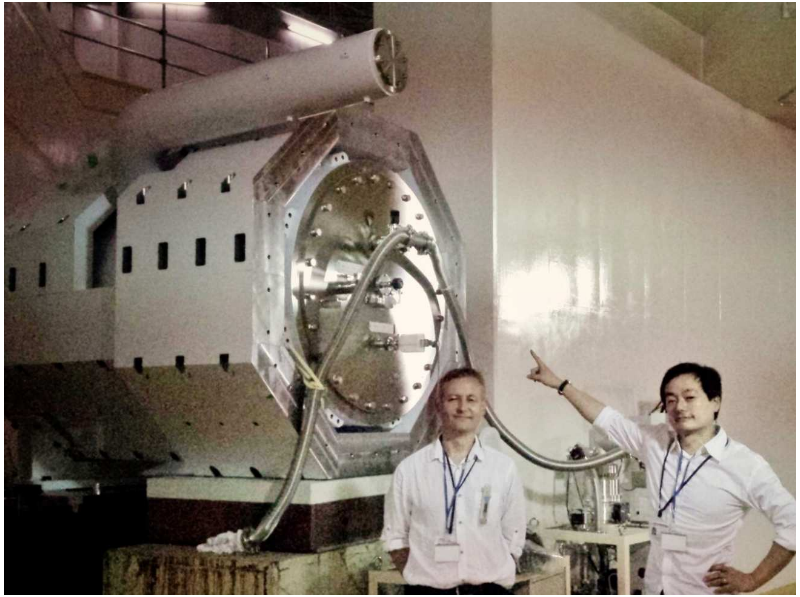
- Background: COMET Project
- Theory: Bethe-Bloch Formula
- Method: Monte-Carlo Simulation
- Results:
  - Impact location distributions
  - Path length distributions
  - p and $\Delta E/\Delta x$ distributions
- Analysis:
  - Fitting methods: Moyal and Landau
  - Fitting of p and $\Delta E/\Delta x$
  - Plot fitted data against Bethe-Bloch formula
- Conclusion



Dr Per Jonsson and Professor Yoshi Uchida at the J-PARC proton accelerator complex in Tokai, Japan.

Background: COMET Project

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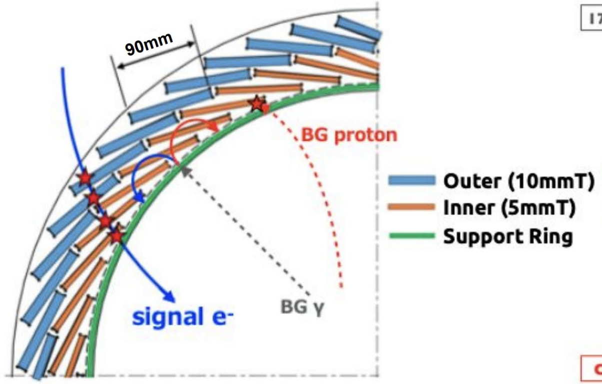
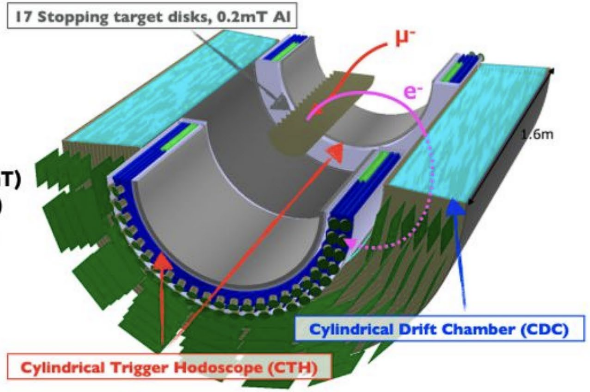
- COMET detects $\mu^- N \rightarrow e^- N$, e^- at 105 MeV/c with a branching ratio of $O(10^{-15})$ [1]
- Investigate CTH detector response upon encountering e^-, mu^+ and pi^+, at momenta of 105, 125, 150 and 200 MeV/c
- Aim to better understand the behaviour of CTH detector response, hence prepare for future data analysis





[2]

[1] R. Abramishvili et al, "COMET Phase-I technical design report," Ptep, vol. 2020(3), (3), pp. 1, Mar 1, 2020.
[2] Y. Fujii, "A search for a muon to electron conversion in COMET," *J. Inst.*, vol. 18, *(10),* 2023. . DOI: 10.1088/1748-0221/18/10/c10010.

Theory: Bethe-Bloch Formula

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- Bethe-Bloch formula predicts energy deposition
- Derived from
  - 1) Inelastic collisions with atomic electrons
  - 2) Elastic scattering from nuclei
- Initially calculated classically by Bohr:

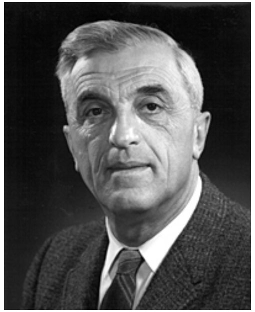
$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_e v^2} N_e \ln \frac{\gamma^2 m v^3}{z e^2 \bar{\nu}} \quad [1]$$

- Then Bethe and Bloch considered QM:

$$-\frac{dE}{dx} = 2\pi N_a r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2 m_e \gamma^2 v^2 W_{max}}{I^2} \right) - 2\beta^2 \right] \quad [1]$$

Niels Bohr
(1885 – 1962)

Hans Bethe
(1906 – 2015)

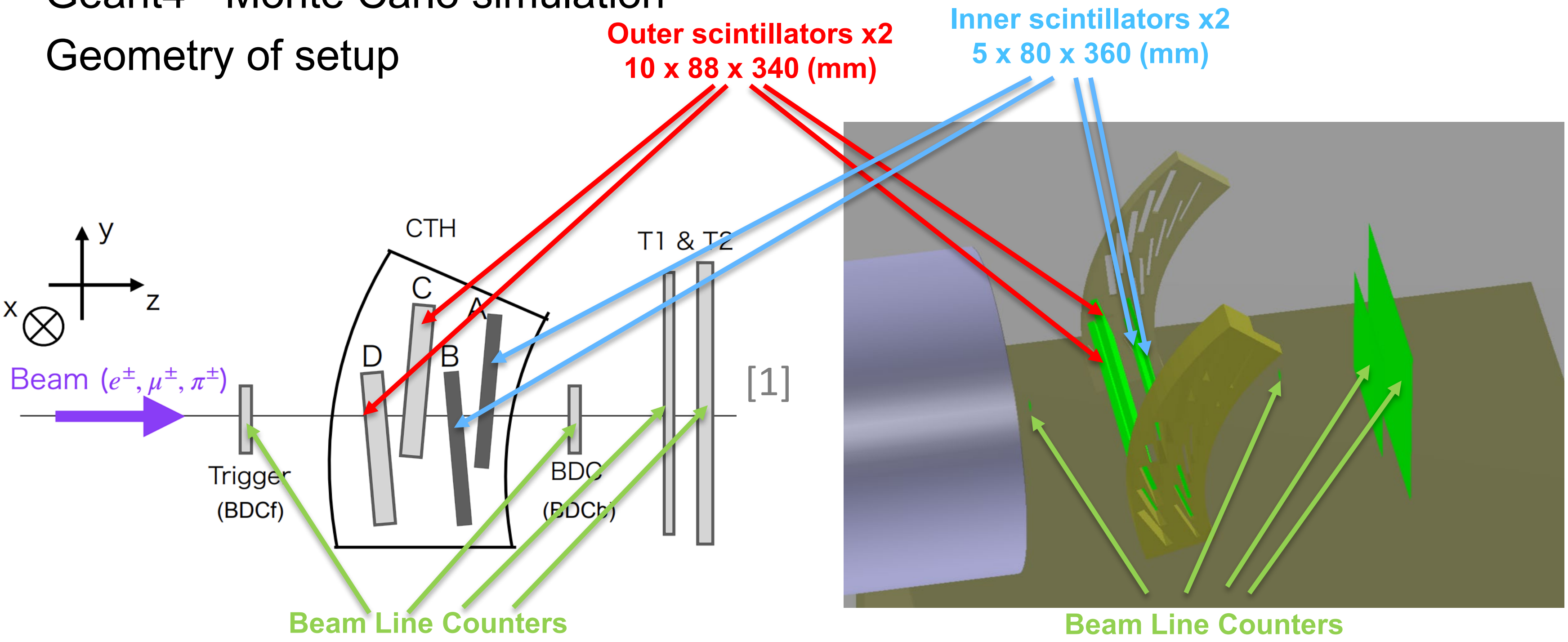


Felix Bloch
(1905 – 1983)

[1] W. R. Leo 1948, Techniques for Nuclear and Particle Physics Experiments : A how-to Approach / William R. Leo. (2nd rev. ed ed.) Tokyo: Springer, 1994.

Method: Monte Carlo simulation

- Geant4 - Monte Carlo simulation
- Geometry of setup



[1] Y. Fujii, "CTH Timing Resolution," unpublished, private communication, Monash University, Jan 2024.

Impact location distributions of 105 MeV/c

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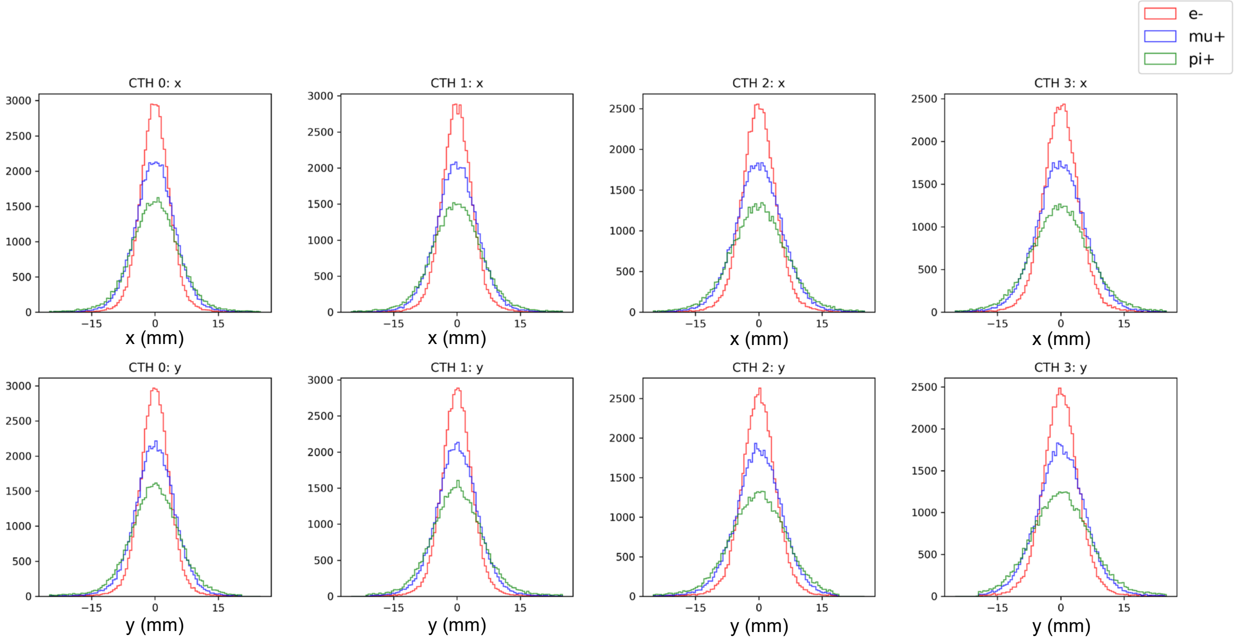
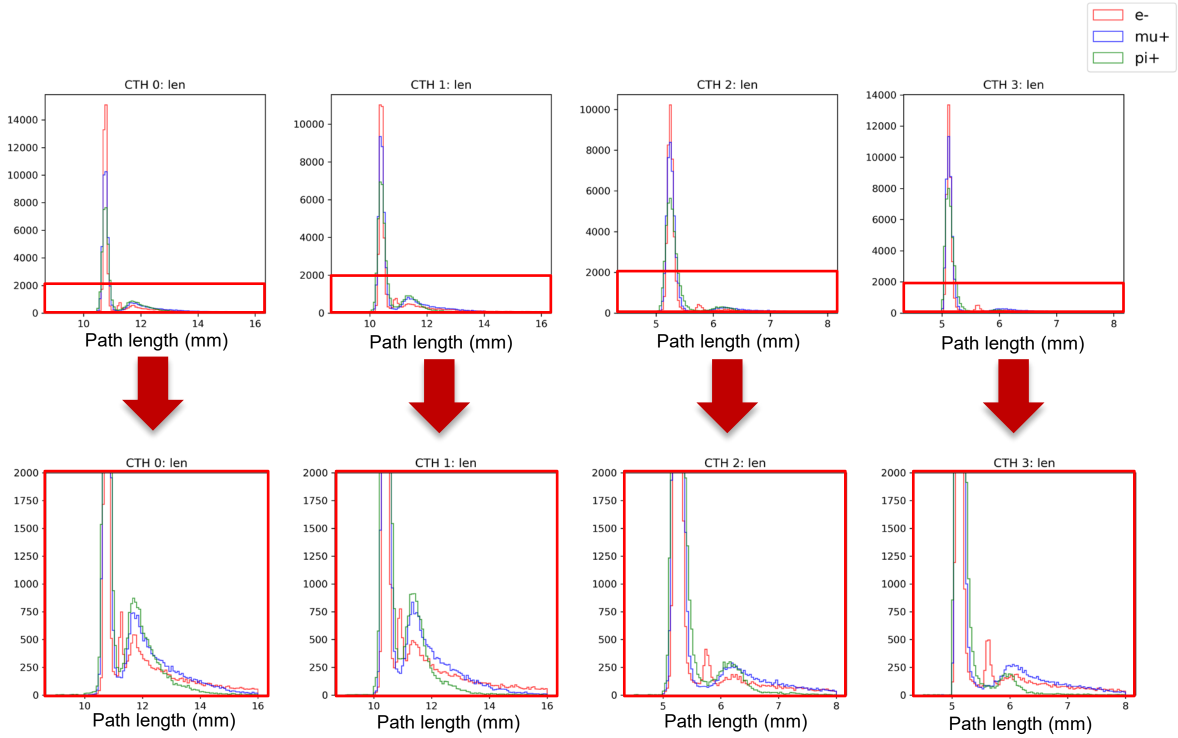
- Data extracted from Geant4 simulation
- x, y distribution graphs
- m_e = 0.511 MeV/c^2
  m_μ = 105.7 MeV/c^2
  m_π = 135.0 MeV/c^2
  $m_\pi > m_\mu > m_e$
- Heavier particles experience stronger scattering
- A result of relativistic effects



Path length distributions of 105 MeV/c

- Path length distributions
- CTH 0: 10 mm
  CTH 1: 10 mm
  CTH 2: 5 mm
  CTH 3: 5 mm
- Small bumps after the major peak
- Potentially resulted from secondary particles



p and ΔE/Δx distributions of 105 MeV/c

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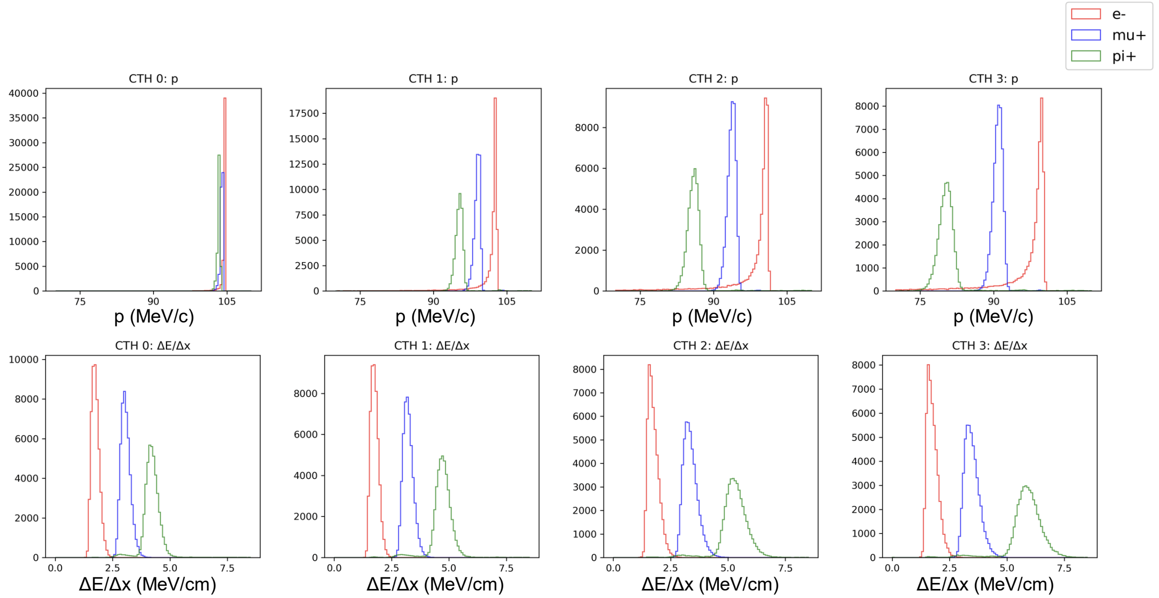
- p distributions
- Heavier particles loses p faster
- ΔE/Δx obtained by dividing edep by len
- Heavier particles deposits more energy
- Results of relativistic effects



Fitting methods: Landau and Moyal

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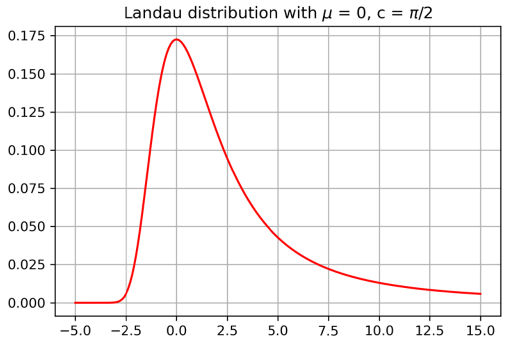
- Landau distribution

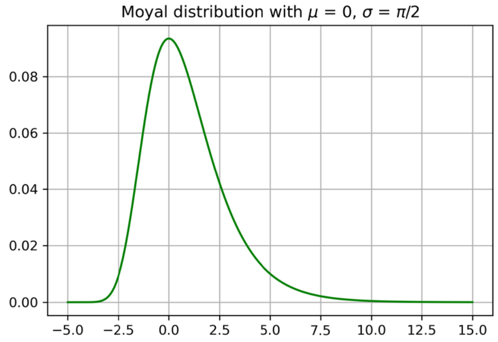
$$p(x;\mu,c) = \frac{1}{\pi c}\int_0^\infty e^{-t}\cos\left(t\left(\frac{x-\mu}{c}\right) + \frac{2t}{\pi}\log\left(\frac{t}{c}\right)\right)dt \text{ [1]}$$

- Moyal distribution

$$p(\lambda,\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{\lambda + e^{-\lambda}}{2}\right), \quad \lambda(x;\mu,\sigma) = \frac{x-\mu}{\sigma} \text{ [2]}$$

- Moyal is a good approximation of Landau when mean number of collisions Q ≥ 20





[1] L. D. Landau, " On the energy loss of fast particles by ionization," J. Phys., vol. 8, (4), pp. 201-205, 1944.
[2] J. E. Moyal, "XXX. Theory of ionization fluctuations," The London, Edinburgh and Dublin Philosophical Magazine and Journal of Science, vol. 46, (374), pp. 263-280, 1955.

Fitting of p and ΔE/Δx of 105 MeV/c

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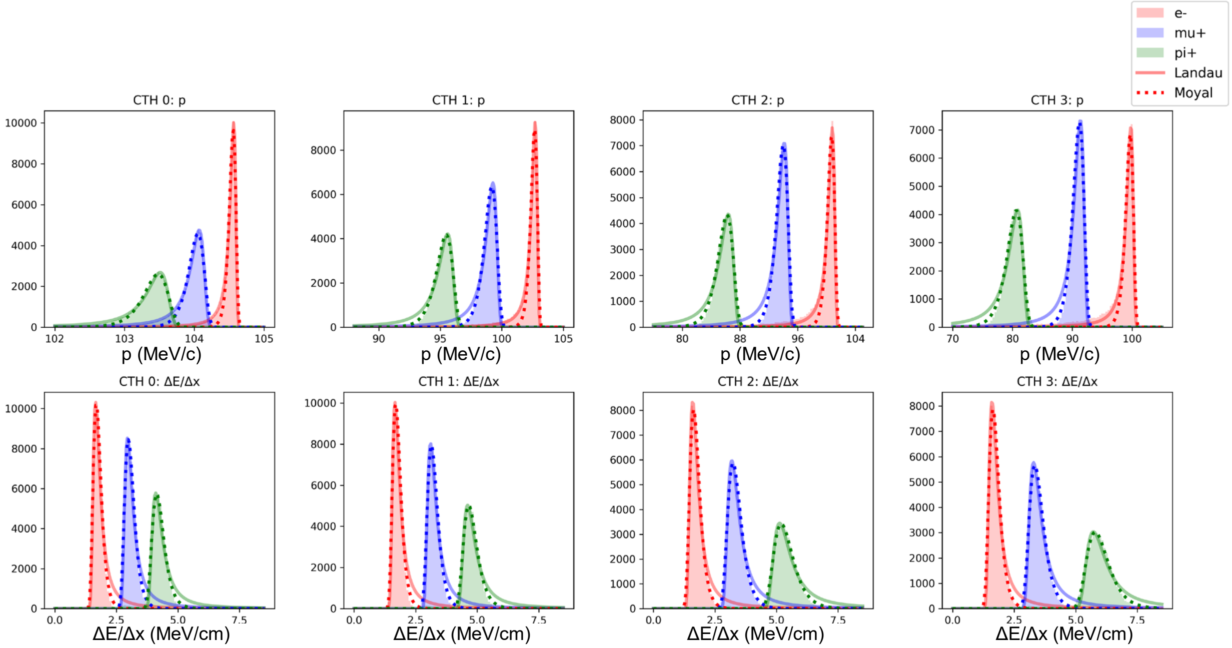
- Since correlation between p & ΔE/Δx, can also fit p
- Rescale the range of p
- Fit Landau distribution
- Fit Moyal distribution



Fitting of p and ΔE/Δx of 105 MeV/c

- Find chi-squared value of each fitting
- Find smaller values between the two fits
- Determine the better fit

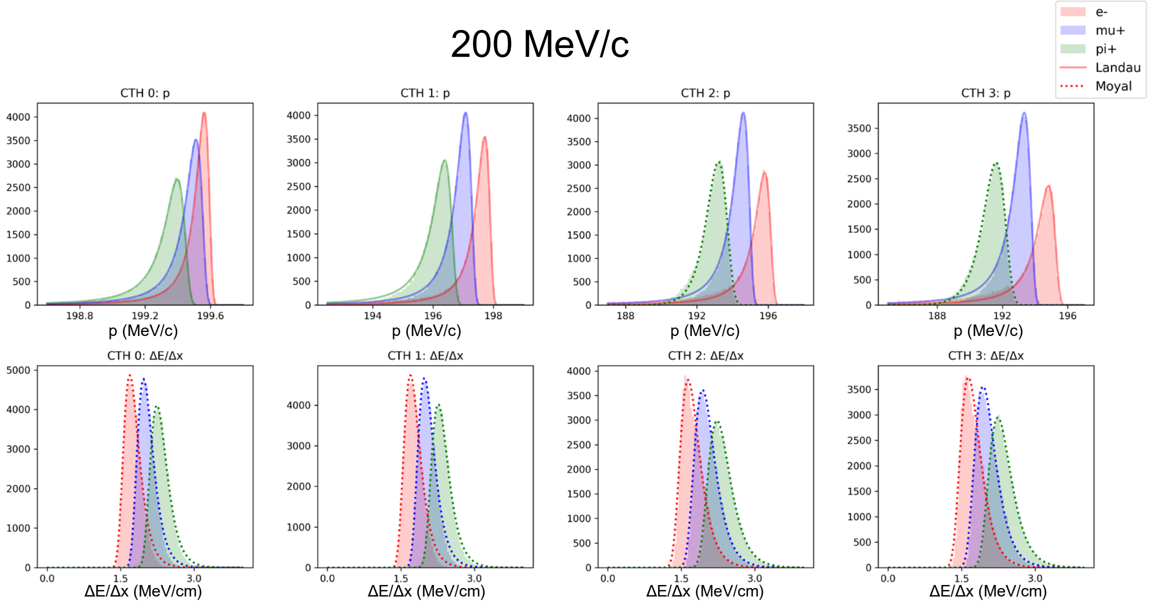
$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

		105 MeV/c											
		CTH 0			CTH 1			CTH 2			CTH 3		
		e-	mu+	pi+	e-	mu+	pi+	e-	mu+	pi+	e-	mu+	pi+
P	Landau	10.21	25.38	60.97	73.98	115.97	244.2	172.14	212.65	495.38	228.36	630.92	1394.66
	Moyal	6174.02	1246.55	111.5	12686.15	9.58	19.89	12279.81	20.21	62.35	11130.81	35.85	128.9
	Better fit	Landau	Landau	Landau	Landau	Moyal	Moyal	Landau	Moyal	Moyal	Landau	Moyal	Moyal
dE/dx	Landau	198.6	280.23	969.75	184.58	703.45	324.91	100.15	191.06	674.48	100.33	191.01	543.17
	Moyal	51.79	51.95	108.12	64.7	60.79	133.19	35.05	10.15	56.33	35.31	12.47	44.43
	Better fit	Moyal	Moyal	Moyal	Moyal	Moyal	Moyal	Moyal	Moyal	Moyal	Moyal	Moyal	Moyal

Fitting of p and ΔE/Δx of all momenta

- Now we have favoured fits
- Fit 105 MeV/c distributions
- With a similar method, fit distributions of 125 MeV/c, 150 MeV/c, and 200 MeV/c

		200 MeV/c			
		CTH 0	CTH 1	CTH 2	CTH 3
p	e-	Landau	Landau	Landau	Landau
	mu+	Landau	Landau	Landau	Landau
	pi+	Landau	Landau	Moyal	Moyal
dE/dx	e-	Moyal	Moyal	Moyal	Moyal
	mu+	Moyal	Moyal	Moyal	Moyal
	pi+	Moyal	Moyal	Moyal	Moyal



- As momentum increases, Landau is favoured by p and distributions merge into each other

Plot fitted data against Bethe-Bloch formula

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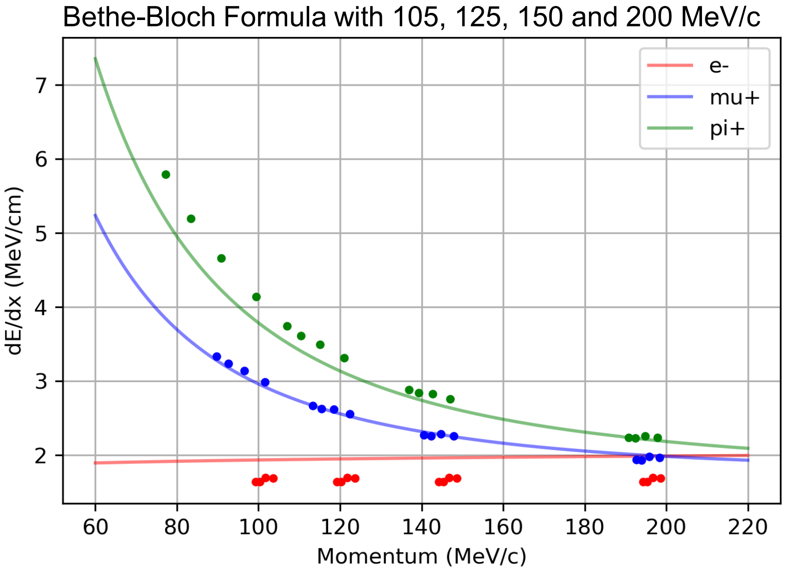
- PVT material: [CH2CH(C6H4CH3)]n [1]
- Find Z_{eff} and A_{eff} using $$Z_{\text{eff}} = \sqrt[2.94]{f_1 \times (Z_1)^{2.94} + f_2 \times (Z_2)^{2.94} + f_3 \times (Z_3)^{2.94} + \cdots}$$ [2]

  => Z_{eff} = 5.665, A_{eff} = 11.6525

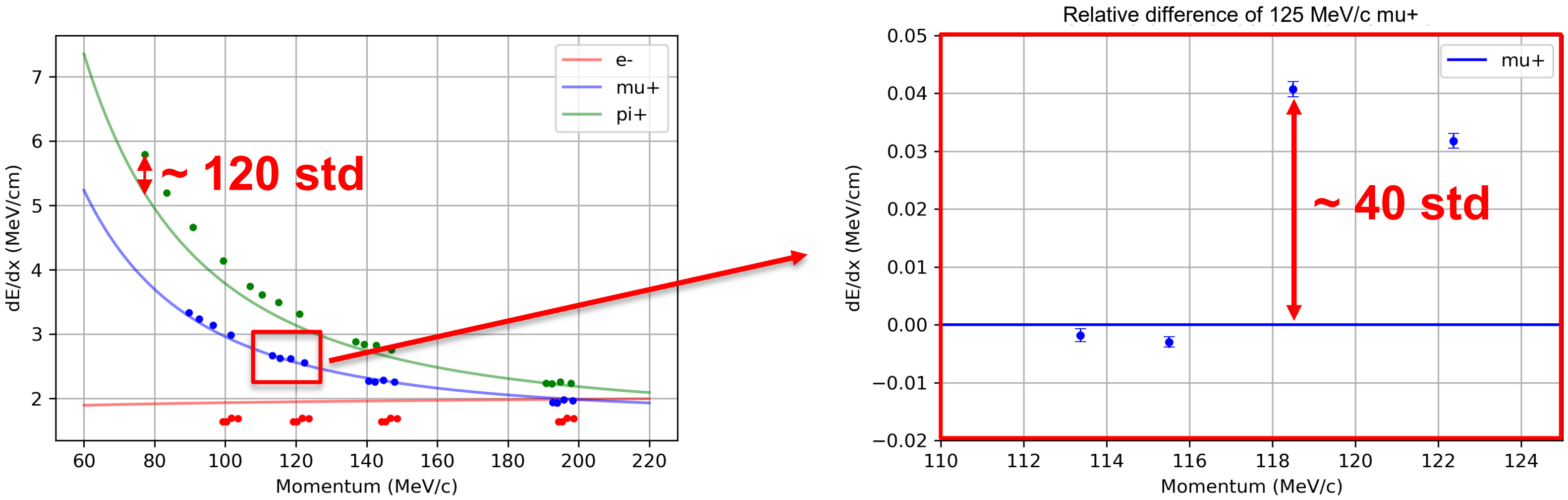
- Hence plot Bethe-Bloch formula
- Plot fitted MPV results with error for 105 MeV/c, 125 MeV/c, 150 MeV/c, and 200 MeV/c
- General trend matches well



[1] J. B. Birks et al, The Theory and Practice of Scintillation Counting. (1st ed.) Kent: Elsevier Science & Technology, 1964.
[2] R. C. MURTY, "Effective Atomic Numbers of Heterogeneous Materials," Nature, vol. 207, (4995), pp. 398-399, 1965.

Plot fitted data against Bethe-Bloch formula

- Error bars not visible
- Error ~ $O(10^{-3})$ from statistical fitting errors
- Must be other potential systematic errors



Conclusion

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- Moyal distribution fits ΔE/Δx well
- For p, lighter and faster particles prefer Landau, while others prefer Moyal
- Bethe-Bloch well predicts the trend of p and ΔE/Δx of different particles
- Large discrepancy needs further investigation
- At 105 MeV/c, COMET's range of interest, e- can be distinguished
- e- can also be a secondary particle resulted from other particles
  => Such cases should also be investigated

Thank You!

Plot fitted data on Bethe-Bloch formula

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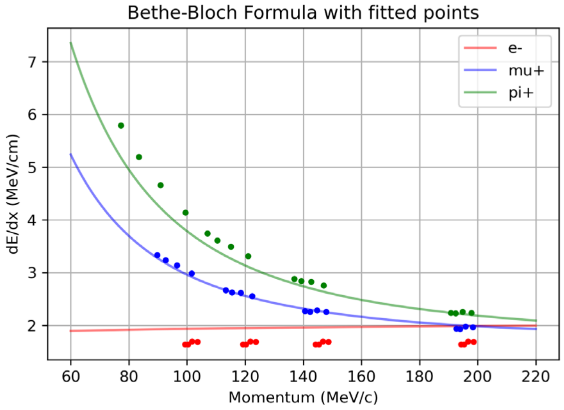
- Fitted MPV results



		200 MeV/c							
		CTH 0		CTH 1		CTH 2		CTH 3	
		MPV	error	MPV	error	MPV	error	MPV	error
p	e-	199.5624	0.0001	197.717	0.001	195.806	0.005	194.817	0.008
	mu+	199.5119	0.0002	197.084	0.001	194.609	0.004	193.338	0.005
	pi+	199.3970	0.0003	196.387	0.002	193.210	0.006	191.606	0.005
dE/dx	e-	1.686	0.001	1.693	0.002	1.638	0.003	1.639	0.003
	mu+	1.968	0.001	1.980	0.001	1.934	0.002	1.938	0.002
	pi+	2.239	0.001	2.259	0.001	2.228	0.001	2.238	0.001

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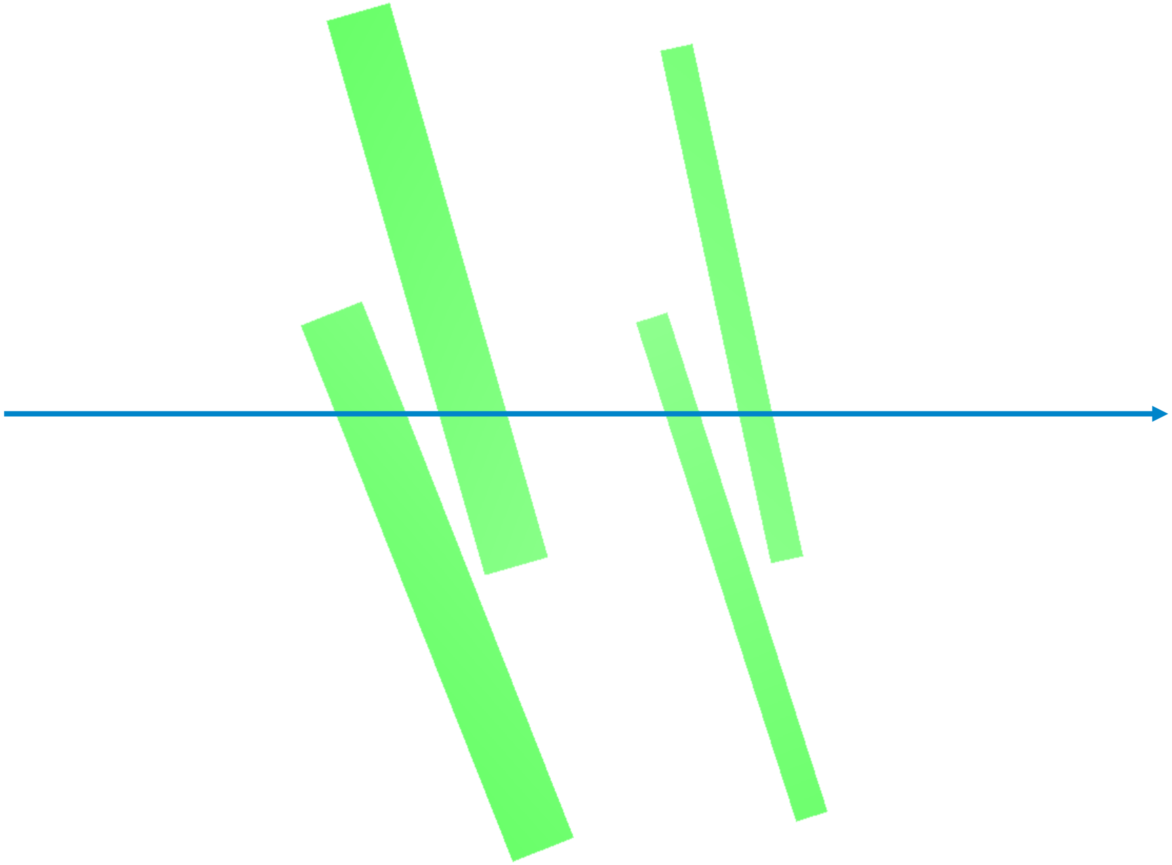

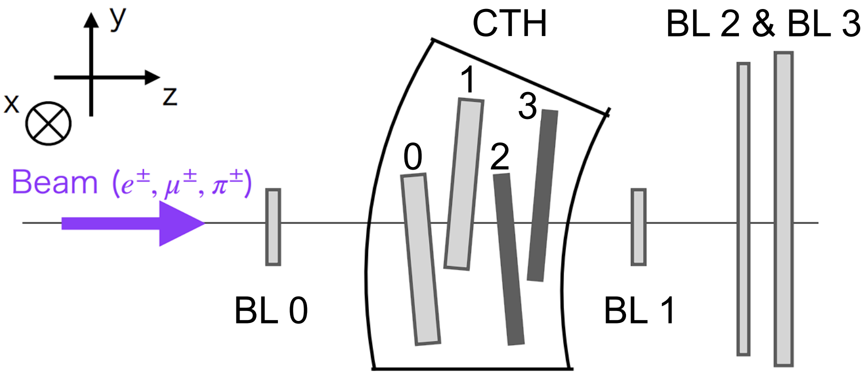
y
z
x
CTH
BL 2 & BL 3
1
3
0
2
Beam $(e^{\pm}, \mu^{\pm}, \pi^{\pm})$
BL 0
BL 1